PHYS 4200

Summary of first half of course

Math
Probability - coins and dice; probability of multiple independent events.
Probability distributions - discrete and continuous, 1 and multiple variables
Moments of distributions - mean, variance (or dispersion), standard deviation
Random walks - probability of n_1 steps right; probability of ending at m steps to right
Binomial distribution - N choose n_1 is $N!/[n_1!(N-n_1)!]$
Stirling's approximation - $\ln N! \sim N \ln N - N$
Gaussian distribution - $p(x) = 1/\sigma\sqrt{(2\pi)} \exp[-(x-\mu)^2/2\sigma^2]$
Poisson distribution - $P(k) = \lambda^k e^{-\lambda}/k!$, mean $= \lambda$, variance about mean $= \lambda$
Gaussian approximation - approximating distribution with Gaussian is often adequate
Central limit theorem - Adding n random variables, with mean μ and variance σ^2,
       approaches Gaussian with mean $n\mu$ and variance $n\sigma^2$.
Converting the independent variable - e.g. from $p(\phi)$ and $\phi(x)$, compute $p(x)$.
Exact and inexact differentials - how to determine, state functions.
Hyperbolic trig functions.

State space
Microstates and macrostates
Classical phase space - e.g. ideal gas, classical harmonic oscillator; partitioning space
Quantum state space - e.g. spins, particle in box, quantum harmonic oscillator
H-theorem - using microscopic reversibility, this shows that H always decreases, where H
       is essentially the negative of the entropy and relates to localization in state space.
Liouville theorem - using master equation for probability flow, shows that probability
       flow in phase space has zero divergence, like incompressible fluid flow.
Principle of equal a priori probabilities - holds if system is at equilibrium
Probability of macrostate - macrostate with property y has probability $\Omega(E;y)/\Omega(E)$.
Reversible and irreversible processes

Density of states
System with f degrees of freedom - $\Omega(E) \sim E^{f/2}$; $E = fkT/2$.
Ideal gas / particle in a box - $\Omega(E) = BV^N E^{3N/2}$; $E = 3/2\ NkT$.
Harmonic oscillators - $\Omega(E) = (E/h\nu)^N$ (for indistinguishable harmonic oscillators)
Spin systems
Polymer

Heat and work
Work - W is work done by system; e.g. $dW = p\ dV$.
Generalized forces - parameter is x and generalized force is X; examples are V and p.
Heat - Q is heat added to the system. $Q = \Delta E + W$.
Conservation of energy - $\Delta E = -W + Q$
Quasi-static processes - parameters change more slowly than system relaxation time.

System properties

Thermodynamic β - defined as $\beta = \partial \ln \Omega / \partial E$. Also, $\beta = 1/kT$.

Entropy - defined as $S = k \ln \Omega$. Also $dS = dQ/T$. Entropy is a state function. As T
   decreases to 0, S decreases to S_0 (often 0). Also, $\Delta S = \int dQ/T = \int C_y(T)/T \, dT$.

Temperature - defined as $T = 1/k\beta$. Also, $1/T = \partial S / \partial E$. Temperature can be positive,
   negative, 0 or ∞.

Interacting systems - at equilibrium, $\beta_1 = \beta_2$, $T_1 = T_2$, and $p_1 = p_2$.

Generalized force - compute using $\beta X = \partial \ln \Omega / \partial x$. Also, $X/T = \partial S / \partial x$.

Equation of state - relate generalized forces, external parameters, and temperature; e.g.
   ideal gas law is $pV = NkT$.

Heat capacity - at constant y, $C_y = (\partial Q / \partial T)_y = T \, (\partial S / \partial T)_y$.

Molar heat capacity - heat capacity per mole, given as c_y.

Laws of thermodynamics

0th - if two systems are in equilibrium with a third, then they are in equilibrium with each
   other.

1st - Energy is conserved. If a system interacts with others, $\Delta E = -W + Q$.

2nd - Total entropy always stays the same or increases in any process; $dS = dQ/T$.

3rd - As temperature decreases to 0, S decreases to S_0, which is a constant.