Homework 7

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7.1 Consider a homogeneous mixture of inert monatomic ideal gases at absolute temperature T in a container of volume V. Let there be ν_1 moles of gas 1, ν_2 moles of gas 2, ..., and ν_k moles of gas k.
(a) By considering the classical partition function of this system, derive its equation of state, i.e. find an expression for its total mean pressure $\bar{p}$.
(b) How is this total pressure $\bar{p}$ of the gas related to the pressure $\bar{p}_i$ which the ith gas would produce if it alone occupied the entire volume at this temperature? (This is called the partial pressure of gas i.)

7.3 A thermally insulated container is divided by a partition into two compartments, the right compartment having a volume b times as large as the left one. The left compartment contains ν moles of an ideal gas at temperature T and pressure $\bar{p}$. The right compartment also contains ν moles of an ideal gas at the temperature T. The partition is now removed (without work being done in the process). Calculate:
(a) the final pressure of the gas mixture in terms of $\bar{p}$;
(b) the total change of entropy if the gases are different;
(c) the total change of entropy if the gases are identical.

7.5 (Edited from Reif.) A rubber band at absolute temperature T is fastened at one end to a peg, and supports from its other end a weight that exerts force W. Assume as a simple microscopic model of the rubber band that it consists of a linked polymer chain of N segments joined end to end; each segment has length $b = \frac{L}{N}$, where L is the total pathlength of the polymer, and can be oriented either parallel or antiparallel to the vertical direction.
(a) Find an expression for the mean length x of the rubber band as a function of W. (Neglect the kinetic energies or weights of the segments themselves, or any interaction between the segments.)
(b) Invert the above expression to express the force W as a function of the extension x. Compare your result to the answer to problem 5 of Homework 4.
(c) What is the Hooke's law spring constant in the limit that $x << L$?

7.6 Consider a gas which is *not* ideal so that the molecules *do* interact with each other. This gas is in thermal equilibrium at the absolute temperature T. Suppose that the translational degrees of freedom of this gas can be treated classically. What is the mean kinetic energy of (center-of-mass) translation of a molecule in this gas?

7.7 Monatomic molecules adsorbed on a surface are free to move on this surface and can be treated as a classical ideal two-dimensional gas. At absolute temperature T, what is the heat capacity per mole of molecules thus adsorbed on a surface of fixed size?

7.9 A very sensitive spring balance consists of a quartz spring suspended from a fixed support. The spring constant is α, i.e. the restoring force of the spring is $-\alpha x$ if the spring is stretched by an amount x. The balance is at a temperature T in a location where the acceleration due to gravity is g.
(a) If a very small object of mass M is suspended from the spring, what is the mean resultant elongation $\bar{x}$ of the spring?

(b) What is the magnitude $\overline{(x-\bar{x})^2}$ of the thermal fluctuations of the object about its equilibrium position? (Hint: think about the energy of the spring, which is a simple harmonic oscillator.)
(c) It becomes impractical to measure the mass of an object when the fluctuations are so large that $\sqrt{\overline{(x-\bar{x})^2}} = \bar{x}$. What is the minimum mass M which can be measured with this balance?

7.19 A gas of molecules, each of mass m , is in thermal equilibrium at the absolute temperature T. Denote the velocity of a molecule by $\boldsymbol{v}$, its three cartesian components by v_x, v_y, and v_z, and its speed by v. What are the following mean values? (Hint: you do not need to compute any integrals.)
(a) $\langle v_x \rangle$
(b) $\langle v_x^2 \rangle$
(c) $\langle v^2 v_x \rangle$
(d) $\langle v_x^3 v_y \rangle$
(e) $\langle (v_x + b v_y)^2 \rangle$
(f) $\langle v_x^2 v_y^2 \rangle$

7.21 (a) What is the most probably kinetic energy $\tilde{\epsilon}$ of molecules having a Maxwellian velocity distribution?
(b) Is it equal to $\frac{1}{2} m \tilde{v}^2$, where $\tilde{v}$ is the most probable speed of the molecules?

7.25 A spherical bulb 10 cm in radius is maintained at room temperature (300 K) except for one square centimeter which is kept at liquid nitrogen temperature (77 K). The bulb contains water vapor originally at a pressure of 0.1 mm of mercury. Assuming that every water molecule striking the cold area condenses and sticks to the surface, estimate the time required for the pressure to decrease to 10^{-6} mm of mercury.

7.30 The molecules of a monatomic ideal gas are escaping by effusion through a small hole in a wall of an enclosure maintained at absolute temperature T.
(a) By physical reasoning (without actual calculation) do you expect the *mean* kinetic energy $\bar{\epsilon}_0$ of a molecule in the effusing beam to be equal to, greater than, or less than the mean kinetic energy $\bar{\epsilon}_i$ of a molecule within the enclosure?
(b) Calculate $\bar{\epsilon}_0$ for a molcule in the effusing beam. Express your answer in terms of $\bar{\epsilon}_i$.